Please attempt the problems on your own before consulting the answer key!

1. **The managing partner of a business is to receive a bonus equal to 20% of the profit after deducting the bonus. This year, the profit before deducting the bonus is $\$18,000$. Find the amount of the bonus, in dollars.**

Answer: $\$3000$

Solution: Let's start by translating English to math. The bonus is equal to 20% of the profit after deducting the bonus, so $b = \frac{20}{100}(p - b)$, where p is the profit and b is the bonus. The problem gives us the profit before deducting the bonus, which is just the profit, so $p = \$18,000$. Now that we have p, we can solve for b, the amount of bonus:

- $b = \frac{20}{100}(p - b)$ | original equation
- $b = \frac{1}{5}(18000 - b)$ | substituting in $p = 18000$ and simplifying $\frac{20}{100} = \frac{1}{5}$
- $5b = 18000 - b$ | multiplying both sides by 5
- $6b = 18000$ | adding b on both sides
- $b = 3000$ | solving for b

Thus, the amount of bonus in dollars is our answer, $\$3000$.

2. **Every day, an engineer commutes by train to the city where he works. At $8{:}30$ am, as soon as he gets off the train, a car arrives to pick him up and take him to the plant. One day, the engineer takes a train arriving earlier, and starts walking toward the plant. He meets the car on its way to pick him up. He gets in, and he arrives at the plant 10 minutes earlier than he usually arrives. What time does the engineer meet the car? Present your answer as time in hh:mm format.**

Answer: $8{:}25$

Solution: Ok, let's write down what we know before we start trying to solve it. On a normal day, the engineer gets off the train at $8{:}30$, and the car arrives to pick him up at the same time of $8{:}30$. Then, one day he arrives earlier, at some time $t < 8{:}30$ and walks until he meets the car meant to pick him up, and then arrives at the plant 10 minutes earlier. Let's think about this intuitively. The man walks some distance from the train station, and then meets the car. Since the engineer meets the car on its way to pick him up, we can assume that the car is approaching in a straight line from the plant. Normally the car would have to drive an extra distance past the man to the train station and back to the man, but because the man met the car instead of waiting for it, he arrives 10 minutes earlier. So, the distance to the train station and back to the man takes 10 minutes to travel, 5 minutes to the station and 5 minutes back to the man (we can assume that the speed of the car is constant). Since the car met the engineer when it still had 5 minutes left to reach the station, which he planned to reach at $8{:}30$, we can conclude that the car met the engineer 5 minutes before $8{:}30$, $8{:}25$. The question requests the answer in the format hh:mm (hh and mm are hours and minutes respectively), which our answer is already in (there is no need to put a leading zero). Thus, the time the engineer meets the car is our answer, $8{:}25$.

3. **If it is known that log $2 = a$ and log $17 = b$, find $\log_5(5.78)$. Express your answer in terms of only a and b.**

Answer: $\frac{a+2b-2}{1-a}$

Solution: This question is basically locked behind algebra 2 knowledge, so don't feel bad if you couldn't get it because of that. Moving on, this question is all about algebra manipulation. Keep in mind that while algebraically solving the equation, you should be trying to convert terms into log 2 (a), log 17 (b), and log 10 ($\log 10 = 1$) With that in mind, let's do some algebraic manipulation:

- $\log_5(5.78)$ | original expression

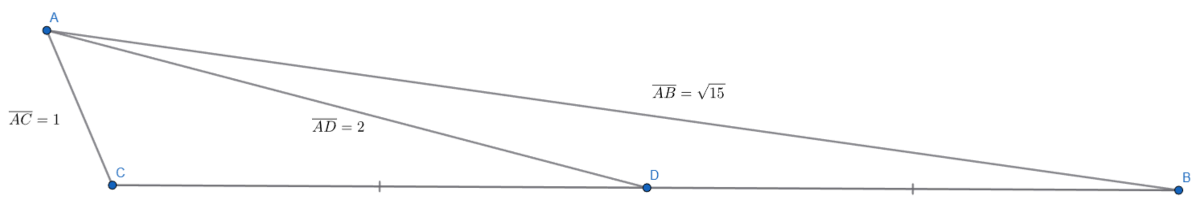
- $\log_5(\frac{2*17^2}{100})$ | substituting $\frac{2*17^2}{100}$ for 5.78
    - During the competition, you will have to notice this. One way to do this is to prime factorize the number. First, make it an integer by multiplying by 100: $5.78 * 100 = 578$. Now notice that it is even, so we can factor out 2: $578 = 2 * 289$. Now notice that 289 is 17^2, so $578 = 2 * 17^2$. Two and seventeen are prime numbers, so we are done factoring. Now just divide by 100 because we multiplied by 100 in the beginning: $5.78 = \frac{2*17^2}{100}$.
- $\frac{log(\frac{2*17^2}{100})}{log(5)}$ | change of base formula
- $\frac{log(\frac{2*17^2}{100})}{log(\frac{10}{2})}$ | turning 5 into $\frac{10}{2}$ because we don't know log 5, but we do know log 10 and log 2.
- $\frac{log(2)+log(17^2)-log(100)}{log(10)-log(2)}$ | expanding the logs
- $\frac{log(2)+2log(17)-2}{1-log(2)}$ | evaluating log(100) and log(10), and bring down power of log(17^2)
- $\frac{a+2b-2}{1-a}$ | substituting $log(2) = a$ and $log(17) = b$

Thus, our answer is $\frac{a+2b-2}{1-a}$.

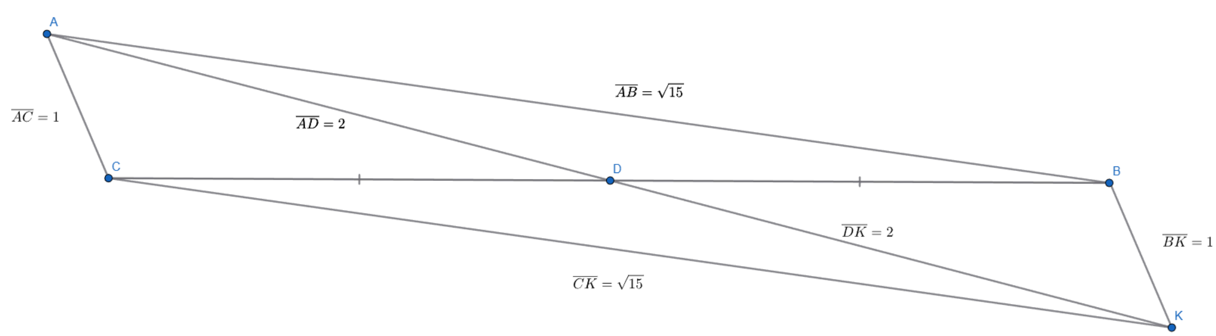
4. **Find the area of a triangle given that two of its sides are 1 and $\sqrt{15}$, and the median dropped to the third side is 2.**

Answer: $\frac{\sqrt{15}}{2}$

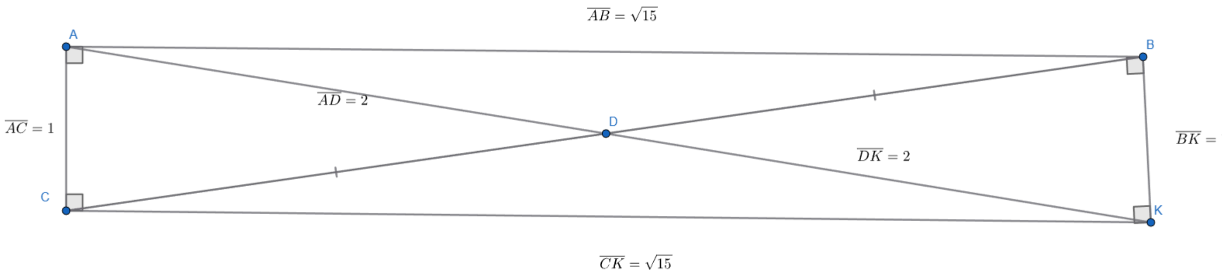
Solution: Let's draw a diagram. Here's the diagram that I eyeballed:



It's not to scale, but as long as we trust the process, it'll be fine. Note that the median of a triangle bisects (splits in half) the other side of the triangle, which I've notated with the dashes on $\overline{CD}$ and $\overline{DB}$, and also splits the enter triangle into two smaller triangles with equal areas (area of ΔACD = area of ΔADB). Now comes the first step, which is to extend $\overline{AD} = \overline{AE}$ and create parallelogram $ABEC$. Note that $\overline{AC} = \overline{BE}$ and $\overline{AB} = \overline{CK}$ by nature of the parallelogram.



Now notice that $\overline{AC} = 1$, $\overline{AB} = \sqrt{15}$, and $\overline{AK} = 4$, forming a pythagorean triple ($1^2 + (\sqrt{15})^2 = 4^2$). Therefore, the angle opposite the hypotenuse, $\overline{AK}$ (it's the longest side), is a right angle. Since $ABEC$ is a parallelogram, if one angle is a right angle, all angles will be right angles. Therefore, let's redraw our parallelogram so that all its interior angles are $90°$:



Now we can see our parallelogram was also a rectangle. We can now solve for the area of the original triangle, ΔABC: $\frac{1}{2}bh = \frac{1}{2}(\sqrt{15})(1) = \frac{\sqrt{15}}{2}$. Thus, our answer is $\frac{\sqrt{15}}{2}$.

5. **In a sequence $\{x_n\}$, it is known that $x_1 = 20$, $x_2 = 17$, $x_{n+1} = x_n - x_{n-1}$ $(n \geq 2)$. Find x_{2025}.**

Answer: -3

Solution: Since we are solving for a very high term number, there's probably a pattern that we can find so that we don't have to manually calculate every term up to x_{2025}. To begin, I'll subtract 1 from each x index in the equation, purely for readability purposes. Not that this will also mean that $n \geq 3$ now:

$$x_n = x_{n-1} - x_{n-2}\ (n \geq 3)$$

Now we can clearly see that the n-th term is the difference of the two previous terms (for $n \geq 3$). Let's solve for x_3:

- $x_n = x_{n-1} - x_{n-2}$ | original equation
- $x_3 = x_{3-1} - x_{3-2}$ | inputting $n = 3$ to solve for x_3
- $x_3 = x_2 - x_1$ | simplifying
- $x_3 = 17 - 20$ | substituting $x_2 = 17$ and $x_1 = 20$ (both are given in the problem)
- $x_3 =- 3$ | simplifying

Let's list out the next few terms. I won't show how to find them, as it's just the same process as above with different values of n:

x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8
20	17	-3	-20	-17	3	20	17

Once we get to term x_8, we can tell that the sequence repeats because the sequence depends on the last two consecutive terms, and we've found the same two consecutive terms as x_1 and x_2. There are 6 terms before the sequence repeats, so the n-th term is equal to $x_{n\,\%\,6}$. Let's check this with $n = 8$ just to make sure:

$$x_8 = x_{8\,\%\,6} = x_2 = 17$$

Great, it works! Now to find $2025 \% 6$, we just long divide 2025 by 6 and look at the remainder which is 3 ($2025 \equiv 3 \pmod 6$)). Therefore:

$$x_{2025} = x_{2025 \% 6} = x_3 = -3$$

The 2025th term in the sequence is -3. Thus, our answer is -3.

6. **Find all values of a, such that the $\begin{cases} x^2 + y^2 = z \\ x + y + z = a \end{cases}$ system has one real solution.**

Answer: $-\frac{1}{2}$

Solution: Ok, we have have what z is, so let's plug it into the second equation and see what we can do:

$$x + y + x^2 + y^2 = a$$

This looks like the equation of a circle. Let's do some algebra to put it in standard form:

- $x + y + x^2 + y^2 = a$ | original equation
- $x + x^2 + y^2 + y = a$ | rearranging terms
- $x^2 + x + (\frac{1}{2})^2 + y^2 + y + (\frac{1}{2})^2 = a + (\frac{1}{2})^2 + (\frac{1}{2})^2$ | completing the square for both x and y
- $(x + \frac{1}{2})^2 + (y + \frac{1}{2})^2 = a + \frac{1}{2}$ | factoring and simplifying

Let's interpret what we have. It's a circle centered at $(-\frac{1}{2}, -\frac{1}{2})$ with a radius of $\sqrt{a + \frac{1}{2}}$. In order for a circle to have only one solution, it must be a single point, which only occurs when the radius is 0. Therefore:

- $\sqrt{a + \frac{1}{2}} = 0$ | original equation
- $a + \frac{1}{2}$ | squaring both sides
- $a = -\frac{1}{2}$ | isolating a

Thus, our answer is $-\frac{1}{2}$

7. **Find all three-digit numbers $\overline{LOM}$, consisting of different digits L, O, and M, for which $(LOM) = (L + O + M)^2 + L + O + M$.**

Answer: 156

Solution: Ok, so we know that $LOM = (L + O + M)^2 + L + O + M$. Factoring, we get:

$$LOM = (L + O + M)(L + O + M + 1)$$

Letting $x = L + O + M$, we get:

$$LOM = x(x + 1)$$

Since LOM is a three digit number, $1000 > LOM > 99$, and by extension $1000 > x(x + 1) \geq 100$. Let's solve to find the range for x:

- $1000 > x(x + 1)$
- $1000 > x^2 + x$
- $x < 32$
    - To find this, I plugged in integer values of x that were integers until the right side was greater than 1000 (which was at $x = 32$)
- $x(x + 1) > 99$
- $x^2 + x > 99$
- $x > 9$

- To find this, I did the same process except I went backwards from 10 until the left side was less than 99, which was at $x = 9$.

Ok, now we know that $32 > x > 9$. We can further constrict x because we know that the sum of the three digits can be at most $9 + 8 + 7 = 24$ (because they are distinct integers 0 to 9). Our new range is $24 \geq x > 9$. Let's continue trying to constrict x:

- $LOM = x(x + 1)$
- $100L + 10O + M = x^2 + x$
- $100L + 10O + M = x^2 + L + O + M$
- $99L + 9O = x^2$
- $3^2 * 11L + 3^2 * O = x^2$
- $3^2(11L + O) = x^2$
- $3\sqrt{11L + O} = x$

This indicates that x is divisible by 3 (because x is an integer). That means x must be $12, 15, 18, 21,$ or 24. Now we can brute-force to see if any of them work:

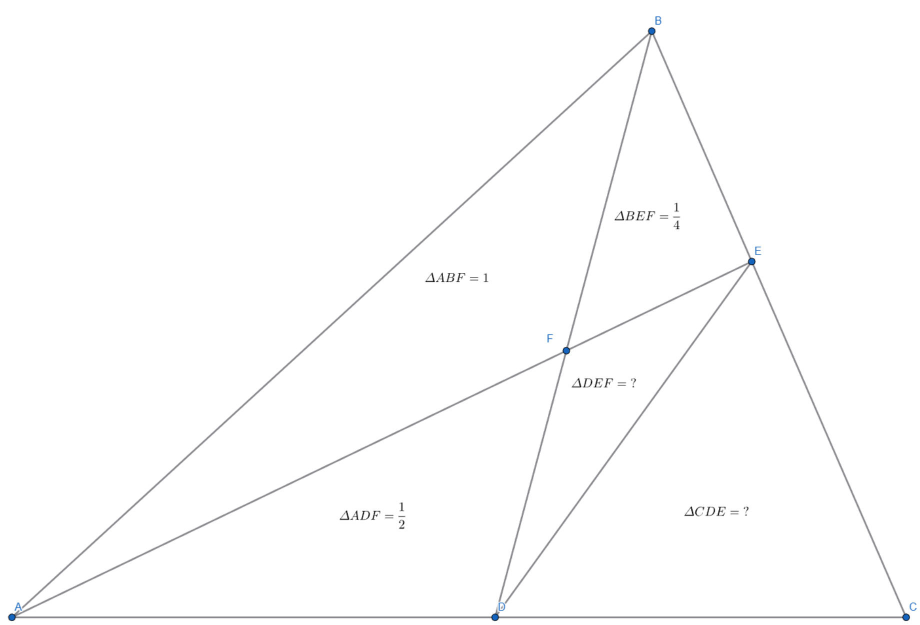
- $12(12 + 1) = 12(13) = 156,\ 1 + 5 + 6 = 12$
- $15(15 + 1) = 15(16) = 240,\ 2 + 4 + 0 \neq 15$
- $18(18 + 1) = 18(19) = 342,\ 3 + 4 + 2 \neq 18$
- $21(21 + 1) = 21(22) = 462,\ 4 + 6 + 2 \neq 21$
- $24(24 + 1) = 24(25) = 600,\ 6 + 0 + 0 \neq 24$

Thus, the only solution is 156. Our answer is 156.

8. **In triangle ABC, points D and E are located on $\overline{AC}$ and $\overline{BC}$, respectively. $\overline{AE}$ and $\overline{BD}$ intersect at F. Find the area of triangle CDE, if the areas of triangles ABF, ADF, and BEF are $1, \frac{1}{2}$, and $\frac{1}{4}$, respectively.**

Answer: $\frac{15}{56}$

Solution: Let's make a diagram. As usual, it won't be to-scale but it'll work:



This construction was relatively easy, so I didn't go through the steps, but if you want to see them please ask us! Anyways, let's get to solving it. Since the ratio of areas is equal to the ratio of the bases in triangles with the same height ([here's a simple proof](#)), we can say that $\frac{\Delta BEF}{\Delta DEF} = \frac{\overline{FB}}{\overline{FD}}$, and that $\frac{\Delta ABF}{\Delta ADF} = \frac{\overline{BF}}{\overline{FD}}$. From there, we can say $\frac{\Delta BEF}{\Delta DEF} = \frac{\Delta ABF}{\Delta ADF}$, and with some algebraic rearranging, $\Delta DEF = \frac{\Delta BEF * \Delta ADF}{\Delta ABF}$. Since we know the areas of the three triangles, we can solve for ΔDEF:

$$\Delta DEF = \frac{\Delta BEF * \Delta ADF}{\Delta ABF} = \frac{\frac{1}{4} * \frac{1}{2}}{1} = \frac{1}{8}$$

Now that we've obtained the area of ΔDEF, let's try finding the area of ΔCDE. We can use the same idea as before, except with bases $\overline{DA}$ and $\overline{DC}$: $\frac{\Delta BDA}{\Delta BDC} = \frac{\overline{DA}}{\overline{DC}}$ and $\frac{\Delta EDA}{\Delta EDC} = \frac{\overline{DA}}{\overline{DC}}$. Now let's solve:

- $\frac{\Delta BDA}{\Delta BDC} = \frac{\Delta EDA}{\Delta EDC}$ | original equation
- $\frac{\Delta ABF + \Delta ADF}{\Delta BEF + \Delta DEF + \Delta CDE} = \frac{\Delta ADF + \Delta DEF}{\Delta CDE}$ | breaking down triangles into areas we know
- $\frac{1 + \frac{1}{2}}{\frac{1}{4} + \frac{1}{8} + \Delta CDE} = \frac{\frac{1}{2} + \frac{1}{8}}{\Delta CDE}$ | substituting values we know
- $\frac{\frac{3}{2}}{\frac{3}{8} + \Delta CDE} = \frac{\frac{5}{8}}{\Delta CDE}$ | simplifying
- $\frac{12}{3 + 8\Delta CDE} = \frac{5}{8\Delta CDE}$ | simplifying
- $96\Delta CDE = 15 + 40\Delta CDE$ | cross multiplying
- $56\Delta CDE = 15$ | bring ΔCDE terms to the left side
- $\Delta CDE = \frac{15}{56}$ | dividing by 56

Thus, the area of ΔCDE is our answer, $\frac{15}{56}$.

9. **For every x, where $x \neq 0$ and $x \neq 1$, function f satisfies $(x - 1)f(x) + f(\frac{1}{x}) = \frac{1}{x-1}$. Find $f(\frac{2024}{2025})$.**

Answer: 2025

Solution: We want to try and find $f(x)$ in terms of x, and not have any other f (we don't want it to be recursive, which means its definition depends on itself. For example, number 5 was a recursive function). To make it not recursive, let's replace x with $\frac{1}{x}$ because then we'll have $f(\frac{1}{x})$ and $f(\frac{1}{\frac{1}{x}}) = f(x)$ again, which will allow us to eliminate either variable using a system of equations:

- $(x - 1)f(x) + f(\frac{1}{x}) = \frac{1}{x-1}$ | original equation
- $(\frac{1}{x} - 1)f(\frac{1}{x}) + f(\frac{1}{\frac{1}{x}}) = \frac{1}{\frac{1}{x} - 1}$ | replacing x with $\frac{1}{x}$
- $(\frac{1-x}{x})f(\frac{1}{x}) + f(x) = \frac{x}{1-x}$ | simplifying
- $f(x) + (\frac{1-x}{x})f(\frac{1}{x}) = \frac{x}{1-x}$ | rearranging terms for later

Now both equations have $f(x)$ and $f(\frac{1}{x})$. Although we could solve for either, let's solve for $f(x)$ because that will make our lives easier. In order to solve for $f(x)$, we must eliminate $f(\frac{1}{x})$ so let's make sure it has the same coefficient in both equations so that we can subtract the equations to eliminate $f(\frac{1}{x})$. To do this, we can multiply the original equation by $\frac{1-x}{x}$ so that the $f(\frac{1}{x})$ term now has a coefficient of $\frac{1-x}{x}$:

- $(x - 1)f(x) + f(\frac{1}{x}) = \frac{1}{x-1}$ | original equation

- $(\frac{1-x}{x})(x-1)f(x) + (\frac{1-x}{x})f(\frac{1}{x}) = (\frac{1}{x-1})(\frac{1-x}{x})$ | multiplying by $\frac{1-x}{x}$
- $(\frac{(1-x)(x-1)}{x})f(x) + (\frac{1-x}{x})f(\frac{1}{x}) = \frac{1-x}{x(x-1)}$ | simplifying
-

Now let's subtract the equations from each other:

- $(\frac{(1-x)(x-1)}{x})f(x) + (\frac{1-x}{x})f(\frac{1}{x}) = \frac{1-x}{x(x-1)}$ | original system of equations

  $- \qquad f(x) + (\frac{1-x}{x})f(\frac{1}{x}) = \frac{x}{1-x}$
- $(\frac{(1-x)(x-1)}{x} - 1)f(x) = \frac{1-x}{x(x-1)} - \frac{x}{1-x}$ | subtracting the equations (eliminating $f(\frac{1}{x})$)
- $(\frac{-x^2+2x-1}{x} - \frac{x}{x})f(x) = \frac{1-x}{x(x-1)} + \frac{x^2}{x(x-1)}$ | simplifying
- $(\frac{-(x^2-x+1)}{x})f(x) = \frac{x^2-x+1}{x(x-1)}$ | simplifying
- $(\frac{-x^2+x-1}{x})f(x) = \frac{x^2-x+1}{x(x-1)}$ | multiplying both sides by $\frac{x}{x^2-x+1}$
- $-f(x) = \frac{1}{x-1}$ | simplifying
- $f(x) = \frac{1}{1-x}$ | simplifying

Now that we know what $f(x)$ is, let's find $f(\frac{2024}{2025})$:

$$f(\frac{2024}{2025}) = \frac{1}{1-\frac{2024}{2025}} = \frac{1}{\frac{1}{2025}} = 2025$$

Thus, our answer is 2025. (This question was really hard, so don't worry if you didn't understand it completely. Just please make sure to ask us what you don't understand, or if you don't understand anything, ask us to explain it to you.)

10. **Solve for x: $81^{sin^2x} + 81^{cos^2x} = 30$, where $0 \le x < \pi$.**

Answer: $\frac{\pi}{6}, \frac{\pi}{3}, \frac{2\pi}{3}, \frac{5\pi}{6}$

Solution: To be honest, I'd recommend just brute-forcing it. I'll go through the brute-force method first, and then go through the actual solution. For the brute-force method, notice that 81 which is 3^4 is raised to a power and added to another 81 raised to a power to equal 30, which is $3^3 + 3$. Seeing as we're working with sin^2x and cos^2x, it's a fair guess that the powers are going to be $\frac{3}{4}$ and $\frac{1}{4}$ ($81^{\frac{3}{4}} = 27$ and $81^{\frac{1}{4}} = 3$). To check, let's solve:

- $sin^2x = \frac{3}{4}$ | original equation
- $sinx = \pm\frac{\sqrt{3}}{2}$ | square rooting each side

$sinx = \pm\frac{\sqrt{3}}{2}$ at $x = \frac{\pi}{3}, \frac{2\pi}{3}$ (note the domain $0 \le x < \pi$). At both $x = \frac{\pi}{3}$ and $x = \frac{2\pi}{3}$, $cos^2x = \frac{1}{4}$, so $\frac{\pi}{3}$ and $\frac{2\pi}{3}$ are solutions. Now let's also try $\frac{1}{4}$:

- $sin^2x = \frac{1}{4}$ | original equation
- $sinx = \pm\frac{1}{2}$ | square rooting each side

$sinx = \pm\frac{1}{2}$ at $x = \frac{\pi}{6}, \frac{5\pi}{6}$. At both of those x values, $cos^2x = \frac{3}{4}$ so $\frac{\pi}{6}$ and $\frac{5\pi}{6}$ are also solutions. Thus, our answer is $\frac{\pi}{6}, \frac{\pi}{3}, \frac{2\pi}{3}, \frac{5\pi}{6}$.

Now, for the algebraic solution, remember the trig identity $cos^2x + sin^2x = 1$. If we let $y = sin^2x$, then we can say $cos^2x = 1 - sin^2x = 1 - y$, and we get the equation $81^y + 81^{1-y} = 30$. Then:

- $81^{y} + 81^{1-y} = 30$ | original equation
- $81^{y} + \frac{81^{1}}{81^{y}} - 30 = 0$
- $(81^{y})^{2} - 30(81^{y}) + 81^{1} = 0$
- $(81^{y} - 27)(81^{y} - 3) = 0$

Thus, either $81^{y} - 27 = 0$ or $81^{y} - 3 = 0$:

- $81^{y} - 27 = 0$
- $81^{y} = 27$
- $log_{3}(81^{y}) = log_{3}(27)$
- $ylog_{3}(3^{4}) = log_{3}(3^{3})$
- $4y = 3$
- $y = \frac{3}{4}$

Doing the same process for $81^{y} - 3 = 0$, we obtain $y = \frac{1}{4}$. Now just solve $sin^{2}x = \frac{3}{4}$ and $sin^{2}x = \frac{1}{4}$. Since I already showed the process during the brute-force method, I won't repeat it again. To note, the reason I recommend the brute-force method is because it's quick and you can be fairly confident you didn't miss a solution. Anyways, our answer is, once again, $\frac{\pi}{6}, \frac{\pi}{3}, \frac{2\pi}{3}, \frac{5\pi}{6}$.

Note that the competition key seems to have a typo in number 9.

The system of equations should be $\left\{\begin{matrix} (x-1)f(x) + f(\frac{1}{x}) = \frac{1}{x-1} \\ (\frac{1}{x}-1)f(\frac{1}{x}) + f(x) = \frac{x}{1-x} \end{matrix}\right\}$ not $\begin{cases} (x-1)f(x) + f\left(\frac{1}{x}\right) = \frac{1}{x-1} \\ \left(\frac{1}{x}-1\right)f\left(\frac{1}{x}\right) + f(x) = \frac{x}{x-1} \end{cases}$.
While I believe it's a typo, it's unlikely that it actually is because I've never seen a mistake in the answer key before. Nonetheless, I believe that I should warn you just in case.
Competition Answer Key:

Central Jersey Mathematics League Contest

[Solutions]

November 12, 2025

1. The managing partner of a business is to receive a bonus equal to 20% of the profit after deducting the bonus. This year, the profit before deducting the bonus is $18,000. Find the amount of the bonus, in dollars.

Solution: Let $b =$ bonus. $b = .20(18000 - b)$, gives $b = 3000$. **Answer: $3,000.**

2. Every day, an engineer commutes by train to the city where he works. At 8:30 am, as soon as he gets off the train, a car arrives to pick him up and take him to the plant. One day, the engineer takes a train arriving earlier, and starts walking toward the plant. He meets the car on its way to pick him up. He gets in, and he arrives at the plant 10 minutes earlier than he usually arrives. What time does the engineer meet the car? Present your answer as time in hh:mm format.

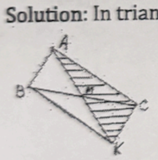
Solution: The car was scheduled to reach the station at 8:30 am. When the car meets the engineer, it saved 10 minutes: 5 minutes to get to the station and 5 minutes to come back to the meeting point. Therefore, the engineer met the car at 8:25 am. **Answer: 8:25 am.**

3. If it is known that $\log 2 = a$ and $\log 17 = b$, find $\log_5(5.78)$. Express your answer in terms of only a and b.

Solution: $\log_5 5.78 = \frac{\log 5.78}{\log 5} = \frac{\log\left(\frac{578}{100}\right)}{\log\left(\frac{10}{2}\right)} = \frac{\log(578) - 2}{1 - \log 2} = \frac{\log(17^2 \times 2) - 2}{1 - a} = \frac{2b + a - 2}{1 - a}$. **Answer:** $\frac{2b + a - 2}{1 - a}$.

4. Find the area of a triangle given that two of its sides are 1 and $\sqrt{15}$, and the median dropped to the third side is 2.

Solution: In triangle ABC (Fig. 1), $AB = 1$, $AC = \sqrt{15}$, and $AM = 2$. Extend $\overline{AM} = \overline{MK}$. Then, $ABKC$ is a parallelogram with $CK = AB = 1$. The area of triangle ABC is equal to the area of triangle ACK, where $AK = 2AM = 4$, $AC = \sqrt{15}$, $CK = 1$. Since $AK^2 = AC^2 + CK^2$ ($16 = 15 + 1$), then triangle ACK is a right triangle with $m\angle ACK = 90°$. It follows that its area $A_{ABC} = A_{ACK} = \frac{1}{2} \cdot AC \cdot CK = \frac{1}{2} \cdot \sqrt{15} \cdot 1 = \frac{\sqrt{15}}{2}$. **Answer:** $\frac{\sqrt{15}}{2}$.



5. In a sequence $\{x_n\}$, it is known that $x_1 = 20$, $x_2 = 17$, $x_{n+1} = x_n - x_{n-1}$ $(n \geq 2)$. Find x_{2025}.

Solution: It follows that $x_{n+3} = x_{n+2} - x_{n+1} = x_{n+1} - x_n - x_{n+1} = -x_n$. Thus, $x_{n+6} = -x_{n+3} = x_n$, that means that the sequence is periodic with a period of 6. Since $2025 = 6 \cdot 337 + 3$, then $x_{2025} = x_3 = -3$. Answer: -3.

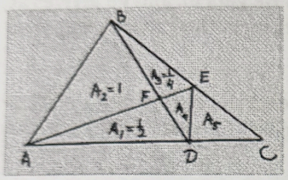
6. Find all values of a, such that the system $\begin{cases} x^2 + y^2 = z \\ x + y + z = a \end{cases}$ has one real solution.

Solution: The system above has one real solution only when $x^2 + y^2 + x + y = a \Leftrightarrow \left(x + \frac{1}{2}\right)^2 + \left(y + \frac{1}{2}\right)^2 = a + \frac{1}{2}$ has one real solution. The set of points on a plane satisfying the above equations is empty if $a < -\frac{1}{2}$, has one point if $a = -\frac{1}{2}$, and is represented by a circle with the center $\left(-\frac{1}{2}, -\frac{1}{2}\right)$ and radius $\sqrt{a + \frac{1}{2}}$ if $a > -\frac{1}{2}$ (in this last case we have infinite number of points). Thus, $a = -\frac{1}{2}$ is the needed solution. **Answer:** $-\frac{1}{2}$.

7. Find all three-digit numbers $\overline{LOM}$, consisting of different digits L, O, and M, for which $(LOM) = (L + O + M)^2 + L + O + M$.

Solution: Let $x = L + O + M$. Then $\overline{LOM} = x(x+1)$. Observe that $x \geq 10$, otherwise $x(x+1) < 100$, and that $x \leq 24$, since the sum of the digits does not exceed $9 + 8 + 7 = 24$. Thus, $x \in [10, 24]$. From $100 \cdot L + 10 \cdot O + M = x^2 + L + O + M$, it follows that $x^2 = 99 \cdot L + 9 \cdot O$, that is x is divisible by 3. Check the numbers 12, 15, 18, 21, and 24, by substituting each number into $x(x+1)$ and calculating the sum of the digits. Only $x = 12$ meets the requirement. Thus, $x = 12$, and $\overline{LOM} = 12 \cdot 13 = 156$. **Answer: 156.**

8. In triangle ABC, points D and E are located on $\overline{AC}$ and $\overline{BC}$, respectively. $\overline{AE}$ and $\overline{BD}$ intersect at F. Find the area of triangle CDE, if the areas of triangles ABF, ADF, and BEF are 1, $\frac{1}{2}$, and $\frac{1}{4}$, respectively.



Solution: Let $A_1 = A_{ADF} = \frac{1}{2}$, $A_2 = A_{ABF} = 1$, $A_3 = A_{BEF} = \frac{1}{4}$, $A_4 = A_{DEF}$, $A_5 = A_{CDE}$ (Fig. 2). Since the ratio of the areas of triangles with equal heights is equal to the ratio of their bases, then it follows $\frac{A_4}{A_3} = \frac{FD}{FB} = \frac{A_1}{A_2}$ (1), $\frac{A_5}{A_3 + A_4} = \frac{CE}{BE} = \frac{A_5 + A_1 + A_4}{A_2 + A_3}$ (2). From (1), we obtain, $A_4 = \frac{A_1 \cdot A_3}{A_2}$, and from (2): $A_5 = \frac{A_1 \cdot A_3 (A_2 + A_3)(A_1 + A_2)}{A_2 (A_2^2 - A_1 \cdot A_3)} = \frac{15}{56}$. **Answer:** $\frac{15}{56}$.

9. For every x, where $x \neq 0$ and $x \neq 1$, function f satisfies $(x-1)f(x) + f\left(\frac{1}{x}\right) = \frac{1}{x-1}$. Find $f\left(\frac{2024}{2025}\right)$.

Solution; Let's substitute $\frac{1}{x}$ instead of x into the original equation. We obtain the following system of linear equations for $f(x)$ and $f\left(\frac{1}{x}\right)$: $\begin{cases} (x-1)f(x) + f\left(\frac{1}{x}\right) = \frac{1}{x-1} \\ \left(\frac{1}{x} - 1\right) f\left(\frac{1}{x}\right) + f(x) = \frac{x}{x-1} \end{cases}$. After subtracting the first equation multiplied by $\frac{1-x}{x}$ from the second equation, we obtain, $\left(1 + \frac{(1-x)^2}{x}\right) f(x) = \frac{x}{1-x} + \frac{1}{x}$. From here, $\frac{x^2 - x + 1}{x} f(x) = \frac{x^2 - x + 1}{x(x-1)}$, or $f(x) = \frac{1}{1-x}$. Thus, $f\left(\frac{2024}{2025}\right) = 2025$. **Answer: 2025.**

10. Solve for x: $81^{\sin^2 x} + 81^{\cos^2 x} = 30$, where $0 \leq x < \pi$.

Solution: Let $\sin^2 x = y$. Then $\cos^2 x = 1 - y$. The original equation becomes $81^y + 81^{1-y} = 30$ or $81^y + \frac{81}{81^y} = 30$. We get a quadratic equation $(81^y)^2 - 30 \cdot 81^y + 81 = 0$. Solving, $81^y = 3$ or $81^y = 27$. Then after solving for y: $\sin^2 x = \frac{1}{4}$ or $\sin^2 x = \frac{3}{4}$. It follows that $\sin x = \pm\frac{1}{2}$ or $\sin x = \pm\frac{\sqrt{3}}{2}$. Then finally solving these two equations for x: Answer: $\frac{\pi}{6}, \frac{5\pi}{6}, \frac{\pi}{3}, \frac{2\pi}{3}$.